Oxford Interviews for Mathematics and Computer Science

Eliot Ball

December 2011

I had four interviews during my time in Oxford. My chosen college was St John's College, where I stayed from the afternoon of Sunday 11 December to the evening of Wednesday 14 December. I had an interview for each half of my subject at St John's, and the same again at St Hugh's, the secondary college to which I was assigned. I was not invited for interviews at any other colleges.

Computer Science at St John's

With Professor Joel Ouaknine and Dr Paul Hunter

The first interview was on the Monday afternoon, with the Computer Science tutors at St John's. At the start of the interview I was offered the opportunity to be considered for the single honours Computer Science degree as well as the joint course with Maths, and I said that I was interested in this. We then immediately began to work on subject-oriented questions. All of my interviews were conducted by two interviewers, and in this interview they each lead half of the interview.

For the first half, I was asked to imagine a tower block with N floors. Floors of the tower block could be travelled to without restriction, and I had a vase which could be thrown from any floor. If the vase was thrown from too high a floor, it would be destroyed, but if the floor was not too high it would not be affected by the fall, and could be thrown again. I was asked how to find the highest safe floor (that is, the highest floor that the vase could be thrown from without it being destroyed) using only the single vase.

I said that if the number of vases was not limited then the best approach would be to throw a vase from half way up the tower, leaving only half of the tower to be considered, and then throw again from half way up this section, and so on until I homed in on the required floor, taking $\log_2 N$ throws in the worst case, but that with only one vase, the best that could be done was to 'walk' up the tower, one floor at a time, observing the last floor from which the vase was thrown without breaking. I justified this by saying that if at any point the vase breaks after skipping a number of floors, it would be impossible to determine which of the skipped floors was the highest safe one. I was asked how many throws this would take in the worst case, and this is obviously N.

The interviewer then asked me what the best strategy would be if I had two vases. My immediate thought was that, with any finite number of vases, it would be best to proceed as if the number of vases was unlimited (binary search) until there was only one remaining, and then walk the final section as before. The interviewer asked if it would still be best to divide the tower in two on the first throw when two vases were available, and I realised that it is not, because the problem is not symmetrical: if the vase did not break then there would be half of the tower remaining, but still two vases. The interviewer suggested that the tower be divided into a number of sections and the first vase would be used to determine which section the highest safe floor was in, with the second being used to find the specific floor. He asked me how many sections the tower should be divided into in order to minimise the number of throws taken in the worst case. After a couple of specific values in order to get a feel for what was going on (it seems obvious in hindsight) I was able to say that the number of throws taken in the worst case, where the tower was divided into k sections, is $T = k - 1 + \frac{N}{k}$. I immediately saw that this could easily be minimised by differentiating: $\frac{dT}{dk} = 1 - \frac{N}{k^2}$. Setting this to zero gives the desired value: $k = \sqrt{N}$.

The second half of the interview, lead by the other interviewer, was with a different problem. I was asked if I had come across graphs before, and I said that I had. The interviewer showed an example directed graph with start and finish vertices, where each edge was labeled with a single letter. Each letter could be used to label more than one edge. The interviewer explained that a graph like this describes a language, with a word being in the language if it is possible to navigate from the start vertex to the end vertex via edges labeled with the required letters. I was first shown a graph describing a language containing the words 'africa' and 'america', and then further graphs where I was asked to describe the languages that they represented. One one graph where the language consisted of any sequence of ’a’s and ’b’s ending with an ’a’, I was quite confused by the interviewer’s way of explaining why this was the language described, but most I was able to quickly work out the set of possible words. We then moved on to a series of languages where I was asked to draw graphs that would result in these languages. I was asked for a graph that allowed any sequence of ’a’s and ’b’s, which is not very difficult, and then for a graph which allowed any sequence of ’a’s and ’b’s of even length. I said that this was equivalent to a graph that would accept any sequence of ’aa’, ’ab’, ’ba’ and ’bb’, and generalised the previous graph, which worked well.

Mathematics at St Hugh’s

Both interviews at St Hugh’s were on the Tuesday afternoon, and I did not know which one would be which until I arrived for the first one, which turned out to be Maths. I was slightly surprised to be interviewed by two women this time, one of whom I think is a postgraduate at the college, because she was definitely much younger than the main interviewer. They were very friendly and seemed to go out of their way to relax me and make me feel comfortable at the start.

The tutor, who was leading the interview, then turned to my reference, where she read out something along the lines of ’Eliot is very confident with complex numbers’. She asked if this was true and then asked me why I like complex numbers. I (perhaps stupidly) said that they have useful applications, which I was asked about, and I said that they could be used in aerodynamics to simplify the modeling of airflow around wings. She asked if I had proved this for myself or whether I had simply been told it, at which point I admitted that it was the latter.

She then asked me if I knew about logs, which I said I did, and she asked me if I could take the log of a complex number. The first thing that came to mind was to use the identity $e^{i\theta} = \cos\theta + i\sin\theta$, from which it immediately follows that $\ln(\cos\theta + i\sin\theta) = i\theta$. I initially thought that the second term in the identity was negative, but the interviewer did not seem to mind giving me the correct form. I said that by solving the part inside the log for θ this would give the required answer. She then pointed out that all of the complex numbers for which this would work were on the unit circle on an argand diagram and asked me if this approach could be adapted to work for any complex number $a + ib$. I said that taking out the modulus of the complex number at the start would allow θ to be found and would leave another term in the answer. Initially the complex number can be expressed in the following form:

$$a + ib = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} + i\frac{b}{\sqrt{a^2+b^2}}\right) = \sqrt{a^2+b^2}\,(\cos\theta + i\sin\theta)$$

Then the log drops out nicely:

$$\ln\left(\sqrt{a^2+b^2}\,(\cos\theta + i\sin\theta)\right) = \ln\sqrt{a^2+b^2} + \ln(\cos\theta + i\sin\theta) = \ln\sqrt{a^2+b^2} + i\theta$$

At the time I apparently had forgotten about FP3 and derived everything, making much more work than necessary. The interviewer said at this point that she believed that I could do complex numbers, and offered the younger woman the opportunity to ask me a question.

She asked me a question that I had seen done before, which is to firstly sketch the graph of $y = \frac{\ln x}{x}$, which I did in the normal way, by considering what happens at extremes, and also differentiating to find the stationary points and which direction the graph is going in at different points. She then asked me to find the pairs of integers x_1 and x_2 such that $x_1 \ln x_2 = x_2 \ln x_1$. I spotted immediately that dividing by $x_1 x_2$ gives an equation in the form of the graph, $\frac{\ln x_2}{x_2} = \frac{\ln x_1}{x_1}$, and this means that the points on the graph with equal y values are needed. Because of the location of the turning point, this means that $1 < x_1 < e$, which means that $x_1 = 2$ and then I said that I could see that $x_2 = 4$.

The initial interviewer then said that I should ask them a question. I asked about her research a little, which was not very interesting since it is related to theoretical physics! Of course I asked some questions as if I was interested. She then said that the time was up and that I had done well, and thanked me for coming.

Computer Science at St Hugh’s

The second interview at St Hugh’s was for Computer Science, and having already had my first Computer Science interview I was feeling confident, because I was expecting more of the same. This was correct, and the interview turned out to have a very similar feel and some of the questions were almost identical.

For the first question I was shown a number of 'tree' diagrams starting with just one node (and no branches) and then increasing the number of rows of branches, adding on twice the number of leaves (end nodes) each time. He asked me how many leaves there were, and I told him by mistake how many nodes there were, $2^n - 1$, where n is the number of rows of nodes. He then reminded me what he asked for, and I told him the correct value for the number of leaves, 2^{n-1}. It's very similar so I would perhaps have got away with it if I had not been explaining what I was doing as I went along!

He then asked me to imagine having eight bags, each of which contains a weight, and asked me to suggest a way to find the heaviest. I initially suggested walking along the row of bags, keeping track of which is heaviest so far, and I said that sorting all of the bags was unnecessary for this, giving the justification that walking the row would take $O(n)$ time whereas the fastest sorting that can possibly be done takes $O(n \log n)$ time. He asked me how many comparisons this would take, and I initially said 8 although it is in fact 7, which I was able to give when the interviewer asked me to make sure. I said that better than 7 cannot be done because every bag must be considered.

The interviewer asked me about finding both the heaviest and lightest bags. I said that as a first attempt the whole process could be carried out twice, which would take 14 steps. The interviewer then suggested that a better solution could be achieved using the trees from the first question. By comparing the bags as if they were leaves on a tree, and following the branches back to the start, the heaviest bag can be determined, still taking 7 steps. However, after the first row of branches have been passed through, the bags are in two sets, one having the heaviest bag as a member, and one having the lightest. The four bags in the latter set can be searched for the lightest bag with just 3 comparisons, putting the total at 10. This was correct, and is the smallest number of steps possible.

For the next question a diagram showing five cards was shown to me. Each card has a number on one side, and a letter on the other side. If the number is even, then the letter has to be a vowel. I was asked how many cards I had to turn over to check that the rules had been followed. The important point is that if the side of the card that is visible has an odd number or a vowel on it, then it does not matter what the other side is and so any such card does not need to be turned over. I missed this initially but it was obvious as soon as the interviewer asked me why I would turn over the first card.

The interview ended with some of the same style graph questions as the first interview had, although perhaps a little harder this time.

Mathematics at St John's

With Professor Charles Batty and Dr David Stirzaker

This interview was on the morning of the last day in Oxford. Although we did not talk about the specific questions, people who had already had this interview earlier in the week had described it as horrible, so I was prepared to find this one a little different. I was not disappointed, and I needed the interviewers to be very involved in giving my ideas throughout the whole interview.

The first things that the interviewers asked me were, I think, intended to settle me in but were delivered in an extremely challenging way that was quite surprising. I was asked what the last maths and non-maths books that I had read were, and why I wanted to come to Oxford.

The interview was entirely mathematical from that point on. The first question that I was asked was to describe the relationship between the sides of a triangle. I suggested a number of obvious things, mainly involving angles, until eventually the interviewer said that they were not necessarily (read: not) looking for an equality. I then said that given any two side lengths a and b of a triangle, and the other side length c then $a + b > c$.

He then asked me to imagine a quadrilateral, and encouraged me to draw it, with the diagonals in. He then asked for a similar relationship between the sum of the diagonals and the sides of the shape. Initially I said that the diagonals each divide the shape into two triangles and that a an upper bound on the sum of the diagonal lengths could be found. If the sides are a, b, c and d, and the diagonals are x and y then we can arrive at $a + b + c + d > x + y$, and at least four more specific inequalities involving fewer letters.

The interviewer said that this gave an upper bound but that they were looking for a lower bound, not necessarily including all sides of the quadrilateral. By considering the four triangles made by the intersecting diagonals, it is possible to show that the sum of the lengths of the diagonals is greater than the sum of the lengths of either pair of opposite sides.

We then moved to an apparently unrelated problem. I was asked to imagine n points in a plane, and asked to suggest a way to find a path that started on some point, visited every point exactly once, and returned to the start point without crossing itself at any point. I suggested a number of useless approaches related to finding the first point when one turns on a point looking outwards and similar, and one thing that I think was quite a good suggestion which is to solve the problem in an inductive way, building up a path from a shorter one, one point at a time, which apparently is hard to do but would work.

The interviewer then suggested that I imagine a path that does have a cross in it, and asked if there was anything I could do about the cross. I had no idea, and he suggested using the result from the first part of the interview. At this point I was quite lost, although I managed to finally see that removing the cross by turning the ’8 shape’ into a ’0 shape’ would produce a shorter path, and this can obviously always be done, because one pair of opposite sides of the quadrilateral around the cross would produce a ’0 shape’ while the other would produce an ’oo shape’. The interview then asked me if this suggested a method for finding a tour that does not cross itself. After what seemed like a very long time the now obvious answer of finding the shortest path occurred to me, which always will produce a path with no crossings. I was asked to say what type of argument we had used to show this, which is a proof by contradiction.

I was then asked why I could be sure that the set of possible touring paths (crossing or not) had a shortest one. I was a little unsure about the purpose of this question and said that there could be numerous shortest paths that have the same length but use different routes. This was not what they were looking for, and asked me for the property of the set that caused it to have a shortest path. I suggested that the set was ’well-ordered’ but this was not what they were looking for either. He said is there a smallest real number and I said no, and this was because there are infintiely many of them, and the answer they were looking for is that the set of possible paths is finite. He then asked me howbig this set was. I managed to not realise that this was the Travelling Salesman Problem and failed to point that out, which is a shame. I said that we had a choice of n points to start the tour on, and then $n-1$ points to visit next, and so on giving $n(n-1)(n-2) \times \cdots \times 3 \times 2 \times 1 = n!$ possible paths. This was not quite the answer they were looking for, since there is only one path when there are 3 points (the triangle described by them) and ignoring different starting points for the same path and the two directions that each path could be taken in the final answer is $\frac{1}{2}(n-1)!$.

I felt that this interview did not go anywhere near as well as the others, and while this may be true, it can’t have gone too badly because I was offered a place at St John’s by letter one week after the day on which this interview took place.

I enjoyed my time at interviews and I would say that even had I been unsuccessful, the interview period would still have been time well spent. I am looking forward to spending more time well over the next four years.

Contact me: eb@eliotball.com